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Scale-invariance and the inertial-range spectrum in three-dimensional stationary, isotropic turbulence

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Abstract
The assumption of scale-invariance, when taken in conjunction with the symmetries of the Navier–Stokes equation, is shown to lead to the energy spectrum $E(k) = \alpha\varepsilon_T^{2/3}k^{-5/3}$, in the limit of infinite Reynolds number. Here ε_T is the flux of energy due to inertial transfer, while the prefactor α is determined by an integral over triple correlations of the phases of the system. It is argued that this form of the prefactor provides an answer to the Landau–Kraichnan criticism of the original Kolmogorov (1941) theory (K41).

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1. Introduction

In 1941, Kolmogorov [1, 2] gave two different derivations of his now-famous result for the second-order structure function

$$S_2 \sim \varepsilon^{2/3} r^{2/3}, \quad (1)$$

for $L > r > l$, where l is a measure of the scale at which viscous effects begin to dominate (i.e. the internal scale) and L is a measure of the large scales of the system (i.e. the external scale). The corresponding result for the energy spectrum in wavenumber is

$$E(k) \sim \varepsilon^{2/3} k^{-5/3}. \quad (2)$$

Shortly after this work was published, it was criticised by Landau (see the footnote on page 126 of [3]; and, for a general discussion, the book by Frisch [4]). Kolmogorov [5] interpreted this criticism as a need to treat the dissipation rate as a variable; and, working with its average taken over a sphere of radius r, concluded that the right-hand side of equation (1) should be multiplied by a factor $(L/r)^{\mu}$, where μ is referred to nowadays as an *intermittency correction*.

This work gave rise to many further attempts by other workers to obtain a value for μ. As a result, for many years K41 has had a question mark hanging over its status as a theory of inertial-range turbulence. For a discussion and references see the review by Sreenivasan [6].

Given the gnomic quality of Landau's criticism of K41, it is of interest to note Kraichnan's interpretation of it [7]. This is to the effect that the universality of the constant of proportionality in equation (1) (or in equation (2)) is prejudiced by the fact that the left-hand side of the equation is an average, whereas the right-hand side is the two-thirds power of an average. That is, denoting instantaneous quantities by a 'hat', we have

$$\langle \hat{E} \rangle \sim \langle \hat{\varepsilon} \rangle^{2/3} k^{-5/3}. \tag{3}$$

In turn, this leads on to a possible dependence on the macrostatistics, by a theory which is supposed to deal with a universal equilibrium of the small scales.

Running counter to the belief in 'intermittency corrections' and 'anomalous exponents', which has been dominant in recent times, there is now a growing view [8–12] that K41 is an asymptotic theory, valid in the limit of infinite Reynolds number. As a result, opinion in the turbulence community is deeply divided on this fundamental issue. Many people seem to find the K41 picture counter-intuitive when one considers aspects of turbulence such as *vortex-stretching*, *localness* and *intermittency*.

2. The basic equations

We introduce the basic equations in k-space. The velocity field $\mathbf{u}(\mathbf{x}, t)$ can be expressed in terms of its Fourier transform $\mathbf{u}(\mathbf{k}, t)$, thus

$$\mathbf{u}(\mathbf{x}, t) \equiv u_\alpha(\mathbf{x}, t) = \int \mathrm{d}^3 k u_\alpha(\mathbf{k}, t) \exp(\mathrm{i}\mathbf{k} \cdot \mathbf{x}). \tag{4}$$

The covariance for homogeneous and isotropic turbulence takes the form

$$\langle u_\alpha(\mathbf{k}, t) u_\beta(\mathbf{k}', t') \rangle = \delta(\mathbf{k} + \mathbf{k}') C_{\alpha\beta}(\mathbf{k}; t, t') = \delta(\mathbf{k} + \mathbf{k}') P_{\alpha\beta}(\mathbf{k}) C(k; t, t'), \tag{5}$$

where the projector $P_{\alpha\beta}(\mathbf{k})$ is expressed in terms of the Kronecker delta as $P_{\alpha\beta}(\mathbf{k}) = \delta_{\alpha\beta} - k_\alpha k_\beta / |\mathbf{k}|^2$. The continuity equation becomes

$$k_\alpha u_\alpha(\mathbf{k}, t) = 0, \tag{6}$$

and the solenoidal Navier–Stokes equation (NSE) becomes

$$\left(\frac{\partial}{\partial t} + \nu k^2 \right) u_\alpha(\mathbf{k}, t) = M_{\alpha\beta\gamma}(\mathbf{k}) \int \mathrm{d}^3 j u_\beta(\mathbf{j}, t) u_\gamma(\mathbf{k} - \mathbf{j}, t), \tag{7}$$

where repeated indices are summed. The inertial transfer operator $M_{\alpha\beta\gamma}(\mathbf{k})$ is symmetric under the interchange of the indices β and γ and is given by

$$M_{\alpha\beta\gamma}(\mathbf{k}) = (2\mathrm{i})^{-1} [\mathrm{k}_\beta \mathrm{P}_{\alpha\gamma}(\mathbf{k}) + \mathrm{k}_\gamma \mathrm{P}_{\alpha\beta}(\mathbf{k})]. \tag{8}$$

It should be noted that the pressure gradient has been eliminated, by using (6) to obtain an equation expressing it in terms of the nonlinearity, and consequently the right-hand side of equation (7) is in solenoidal form (as, trivially, is the left-hand side).

The energy spectrum $E(k, t)$ is related to the spectral density by $E(k, t) = 4\pi k^2 C(k, t)$. The procedure to obtain an equation for this is well known. We first multiply each term in (7) by $u_\sigma(-\mathbf{k}, t)$. Then we form a second equation from (7) for $u_\sigma(-\mathbf{k}, t)$, multiply this by $u_\alpha(\mathbf{k}, t)$, add the two resulting equations together and average the final expression. The resulting equation is

$$\left(\frac{\partial}{\partial t} + 2\nu k^2 \right) P_{\alpha\sigma}(\mathbf{k}) C(k, t) = M_{\alpha\beta\gamma}(\mathbf{k}) \int \mathrm{d}^3 j C_{\beta\gamma\sigma}(\mathbf{j}, \mathbf{k} - \mathbf{j}, -\mathbf{k}; t) \\ - M_{\sigma\beta\gamma}(\mathbf{k}) \int \mathrm{d}^3 j C_{\beta\gamma\alpha}(-\mathbf{j}, -\mathbf{k} + \mathbf{j}, \mathbf{k}; t). \tag{9}$$

Here $C_{\alpha\beta\gamma}(\mathbf{k}, \mathbf{j}, -\mathbf{k}-\mathbf{j})$ stands for the three-velocity correlation. We next set $\sigma = \alpha$, sum over α (noting that $\mathrm{Tr} P_{\alpha\beta} = 2$) and multiply each term in (9) by $2\pi k^2$, to obtain the well-known energy balance equation. We present this later as equation (15). In the meantime, we define the energy transfer spectrum $T(k, t)$ by

$$T(k,t) = 2\pi M_{\alpha\beta\gamma}(\mathbf{k}) \int \mathrm{d}j\, k^2 j^2 \int \mathrm{d}\Omega_j \{C_{\beta\gamma\alpha}(\mathbf{j}, \mathbf{k}-\mathbf{j}, -\mathbf{k}, t) - C_{\beta\gamma\alpha}(-\mathbf{j}, -\mathbf{k}+\mathbf{j}, \mathbf{k}, t)\}, \tag{10}$$

where Ω_j stands for the solid angle traced out by vector $\mathbf{j}$ when the vector $\mathbf{k}$ is chosen as the polar axis. We may establish its properties, as follows.

We consider the inertial transfer of energy in k-space. Write $T(k, t)$ as

$$T(k,t) = \int_0^\infty S(k, j; t)\, \mathrm{d}j, \tag{11}$$

where S depends on the triple moment: its form can be deduced from (10). It can be shown that S is antisymmetric under the interchange $k \rightleftharpoons j$:

$$S(k, j; t) = -S(j, k; t). \tag{12}$$

Hence

$$\int_0^\infty T(k,t)\, \mathrm{d}k = \int_0^\infty \mathrm{d}k \int_0^\infty \mathrm{d}j\, S(k, j; t) = 0 \tag{13}$$

is an exact symmetry which expresses conservation of energy.

The dissipation rate ε_D in k-space is defined by $\varepsilon_D = -\mathrm{d}E/\mathrm{d}t$ for freely decaying turbulence. Integrating over wavenumber, and rearranging, the energy balance becomes

$$\varepsilon_D = -\frac{\mathrm{d}E}{\mathrm{d}t} = \int_0^\infty 2\nu k^2 E(k,t)\, \mathrm{d}k. \tag{14}$$

This is because the inertial transfer term vanishes when integrated over all k. The region in k-space where the dissipation mainly occurs is characterized by the Kolmogorov dissipation wavenumber. An expression for this is given later on as equation (28).

3. Spectral energy transfer and scale-invariance

In this paper, we shall confine our attention to stationary, isotropic turbulence. In turn, this will restrict the concept of universality to mean that spectra are independent of initial conditions or stirring forces. We begin with the energy balance equation

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) = T(k,t) \equiv \int_0^\infty \mathrm{d}j\, S(k, j; t), \tag{15}$$

as derived in the preceding section. For stationarity we must add an input spectrum $W(k)$ (which can be related to the covariance of the random stirring forces [13]). We introduce ε_W as the rate at which the stirring forces do work on the turbulent fluid

$$\varepsilon_W = \int_0^\infty W(k)\, \mathrm{d}k. \tag{16}$$

Then, for stationarity, $\mathrm{d}E(k, t)/\mathrm{d}t \to 0$ and the energy balance becomes

$$T(k) + W(k) - 2\nu k^2 E(k) = 0. \tag{17}$$

Integrating both sides with respect to wavenumber, we have

$$\int_0^\infty W(k)\, \mathrm{d}k - \int_0^\infty 2\nu k^2 E(k)\, \mathrm{d}k = 0; \qquad \text{or} \quad \varepsilon_W = \varepsilon_D. \tag{18}$$

The energy flux is introduced when we integrate each term of (15) with respect to wavenumber, from zero up to some wavenumber κ. Reverting to the general case, using the antisymmetry of S, and making some rearrangements, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^\kappa \mathrm{d}k\, E(k,t) = -\int_\kappa^\infty \mathrm{d}k \int_0^\kappa \mathrm{d}j\, S(k,j;t) - 2\nu\int_0^\kappa \mathrm{d}k\, k^2 E(k,t). \tag{19}$$

In this form the effect of the transfer term is readily interpreted as the net flux of energy from wavenumbers less than κ to those greater than κ, at any time t [14]. Denoting this flux by $\Pi(\kappa)$, and making an exact decomposition of the transfer spectrum into filtered-partitioned forms $T^{+-}(k|\kappa)$ and $T^{-+}(k|\kappa)$ [15], we have

$$\Pi(\kappa) = \int_\kappa^\infty \mathrm{d}k\, T^{+-}(k|\kappa) = -\int_0^\kappa \mathrm{d}k\, T^{-+}(k|\kappa), \tag{20}$$

where we have now assumed stationarity and dropped the time dependence. (Note that the decomposition is completed by $T^{--}(k|\kappa)$ and $T^{++}(k|\kappa)$, which are separately conservative on the intervals $[0,\kappa]$ and $[\kappa,\infty]$, respectively [15].)

The maximum value of the energy flux is $\Pi_{\mathrm{max}}(\kappa)$, where $T^{+-}(k|\kappa) = T^{-+}(k|\kappa) = 0$. At sufficiently large R_λ, the energy containing and dissipation ranges become separated by the inertial range of wavenumbers, thus

$$k_{\mathrm{bot}} \leqslant \kappa \leqslant k_{\mathrm{top}}, \tag{21}$$

where κ now stands for any wavenumber in the inertial range. In this case, the injection and dissipation spectra satisfy approximate relationships as follows:

$$\int_0^{k_{\mathrm{bot}}} \mathrm{d}k\, W(k) \simeq \varepsilon_W; \qquad \text{and} \qquad \int_{k_{\mathrm{top}}}^\infty \mathrm{d}k\, 2\nu k^2 E(k) \simeq \varepsilon_D. \tag{22}$$

Also, in this range of wavenumbers, the maximum energy flux should be approximately constant and we will find it helpful to introduce a specific symbol for this quantity, thus

$$\Pi_{\mathrm{max}} = \varepsilon_T. \tag{23}$$

For stationarity, we must have the overall energy balance

$$\varepsilon_W = \varepsilon_T = \varepsilon_D. \tag{24}$$

We note that these three different physical processes are normally denoted by the single symbol ε, justified by their all being numerically equal. In our view, it is necessary to draw a distinction between them in order to avoid confusion.

To sum this up, we can claim to have a *scale invariant* inertial range of wavenumbers if the following is satisfied:

$$T^{+-}(k|\kappa) = T^{-+}(k|\kappa) = 0; \qquad \text{and} \quad \Pi(\kappa) = \int_\kappa^\infty \mathrm{d}k\, T^{+-}(k|\kappa) = \Pi_{\mathrm{max}}(\kappa) = \varepsilon_T, \tag{25}$$

for constant ε_T, and where κ is any wavenumber in the range given by (21). Recent results suggest that this becomes increasingly true as the Reynolds number is increased to large values [16].

4. The inertial-range energy spectrum

Let us now reconsider K41 in the light of the foregoing discusssion. The first of Kolmogorov's two methods amounted to a statement of similarity principles, along with the use of dimensional analysis. In general, the energy spectrum may depend on any relevant parameter, and for some wavenumber k we may write it as a general functional form

$$E(k) = F[W(k), \Pi(k), \varepsilon_T, k, \nu, \varepsilon_D]. \tag{26}$$

For wavenumbers satisfying the limits in (21), the general form given by equation (26) reduces to

$$E(k) = \alpha \varepsilon_T^{2/3} k^{-5/3} f(k/k_D), \tag{27}$$

where α is the prefactor; and, in the limit of infinite Reynolds numbers, $f \to f(0) = 1$; while the dissipation wavenumber is given by

$$k_D = (\varepsilon_D/\nu^3)^{1/4}. \tag{28}$$

It is usual to replace ε_T in equation (27) by ε_D (actually, by ε). However, the dynamically significant quantity is the inertial flux and we shall leave it as it is, in order to emphasise that fact. We note that in fractal-generated turbulence, the Kolmogorov-like spectrum depends on $(u'^3/L)^{2/3}$, rather than on $\varepsilon_D^{2/3}$ [17]. Also, note that the function f depends on ε_D through the dissipation wavenumber.

4.1. Calculation of the prefactor

Kolmogorov's second method was based on the Karman–Howarth equation, an exact relation connecting the structure functions S_3 and S_2. By setting $\nu = 0$ (its effect is retained through the dissipation rate) he obtained a closed equation for S_3. The result of this *de facto* closure is widely accepted by the turbulence community, but the need to make the additional assumption that the skewness is constant in order to recover (1) is seen as a weakness.

The spectral equivalent of the Karman–Howarth equation is the energy balance given by equation (15). In order to obtain a *de facto* closure, we use scale-invariance, in the form (25), along with equation (10) for $T(k)$, to fix the form of the triple moment, thus

$$2\pi \int_\kappa^\infty \mathrm{d}k\, M_{\alpha\beta\gamma}(\mathbf{k}) \int_0^\kappa \mathrm{d}j\, k^2 j^2 \int \mathrm{d}\Omega_j \{C_{\beta\gamma\alpha}(\mathbf{j}, \mathbf{k}-\mathbf{j}, -\mathbf{k}, t) \\ - C_{\beta\gamma\alpha}(-\mathbf{j}, -\mathbf{k}+\mathbf{j}, \mathbf{k}, t)\} = \varepsilon_T. \tag{29}$$

Now, we introduce a dimensionless form of $u_\alpha(\mathbf{k}, t)$ in the neighbourhood of wavenumber κ, by writing

$$u_\alpha(\mathbf{k}, t) = V(\kappa)\psi_\alpha(\mathbf{k}', t'), \tag{30}$$

where $V(\kappa)$ is the root-mean-square velocity, $\mathbf{k}' = \mathbf{k}/\kappa$, and $t' = t/\tau(\kappa)$. The timescale $\tau(\kappa)$ is to be determined, but is not actually needed for our present purposes.

We will discuss the significance of the function $\psi_\alpha(\mathbf{k}', t')$ presently. For the moment, from equation (30) it follows, by equation (5) and the usual definition of the root-mean-square velocity, that it must satisfy the condition

$$\langle \psi_\alpha(\mathbf{k}', t)\psi_\beta(\mathbf{j}', t)\rangle = P_{\alpha\beta}(\mathbf{k}')\delta(\mathbf{k}' + \mathbf{j}'). \tag{31}$$

Also, it is readily shown that the two-velocity and three-velocity correlations may be expressed in terms of (30) as

$$C_{\alpha\beta}(\mathbf{k}) = P_{\alpha\beta}(\mathbf{k}')\kappa^3 V^2(\kappa), \tag{32}$$

and

$$C_{\alpha\beta\gamma}(\mathbf{k}, \mathbf{j}, -\mathbf{k}-\mathbf{j}) = \kappa^3 V^3(\kappa)\langle \psi_\alpha(\mathbf{k}')\psi_\beta(\mathbf{j}')\psi_\gamma(-\mathbf{k}'-\mathbf{j}')\rangle. \tag{33}$$

This is now the crucial stage of the argument. There are both theoretical and experimental grounds for assuming that the integral in (29) is, when $k = \kappa$ (i.e. in the inertial range), dominated by interactions with $j \sim |\mathbf{k} - \mathbf{j}| \sim k = \kappa$ [14]. However, we can go further than this. We can make use of the exact symmetry

$$S(k, j) = 0 \quad \text{for the case} \quad j = |\mathbf{k} - \mathbf{j}| = k, \tag{34}$$

which corresponds to a zero of the transfer spectrum and hence to a maximum for the flux through wavenumber κ.

4.2. The limit of infinite Reynolds number

Next we take the limit of infinite Reynolds numbers at a constant rate of energy transfer (or dissipation). This implies that the input and output can be concentrated into delta functions in wavenumber, at the origin and infinity, respectively [18]. (Or, for a more accessible reference, see section 6.2.7 of the book [13].) Thus the condition (34) will apply for all wavenumbers. As $u_\alpha(\mathbf{k}, t)$ is complex, we note that $V(\kappa)$ is its amplitude and that $\psi_\alpha(\mathbf{k}', t')$ represents its phase; that is, $\psi_\alpha(\mathbf{k}', t') = \exp[\mathrm{i}\theta_\alpha(\mathbf{k}', \mathrm{t}')]$ [14].

With these points in mind, we substitute equation (33) into (29), transform to scaled variables, invoke stationarity, and integrate over all wavenumbers with k' outside, and j' inside, (say) the unit sphere. In this way, it is readily shown by power counting that the root-mean-square velocity in wavenumber space must take the form

$$V(\kappa) = B^{-1/3}\varepsilon_T^{1/3}\kappa^{-10/3}, \tag{35}$$

where

$$B = 2\pi \int_1^\infty \mathrm{d}k' \int_0^1 \mathrm{d}j' k'^2 j'^2 \int \mathrm{d}\Omega_{j'} M_{\alpha\beta\gamma}(\mathbf{k}')[\langle \psi_\beta(\mathbf{j}', t')\psi_\gamma(\mathbf{k}' - \mathbf{j}', t')\psi_\alpha(-\mathbf{k}', t')\rangle - \langle \psi_\beta(-\mathbf{j}', t')\psi_\gamma(-\mathbf{k}' + \mathbf{j}', t')\psi_\alpha(\mathbf{k}', t')\rangle]. \tag{36}$$

Substituting this result into equation (32), and using (5) for the spectral density $C(k)$, we obtain the inertial range spectrum in the form

$$E(\kappa) = 4\pi\kappa^2 C(\kappa) = \alpha\varepsilon_T^{2/3}\kappa^{-5/3}. \tag{37}$$

The prefactor is now given by $\alpha = 4\pi B^{-2/3}$, and κ is any wavenumber in the inertial range.

5. Conclusion

This would appear to provide an answer to the criticism of K41 by Landau, as interpreted by Kraichnan [7]. Essentially, Kraichnan said that the presence of an average (whether ε_T or ε_D) to the power of $2/3$ on the right-hand side of equation (37) destroys the linearity of the averaging procedure for that equation. According to our present analysis, that dependence is cancelled out by the presence of the factor $B^{-2/3}$.

In conclusion, on the assumption of scale-invariance (in the form set out in equation (29)), the symmetries of the Navier–Stokes equation lead to the '$-5/3$' law for the energy spectrum. Arguably, from the arguments given in [18] (or see [13]), scale-invariance is inevitable in the limit of infinite Reynolds numbers.

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